Exponential Functions $f(x) = b^x$

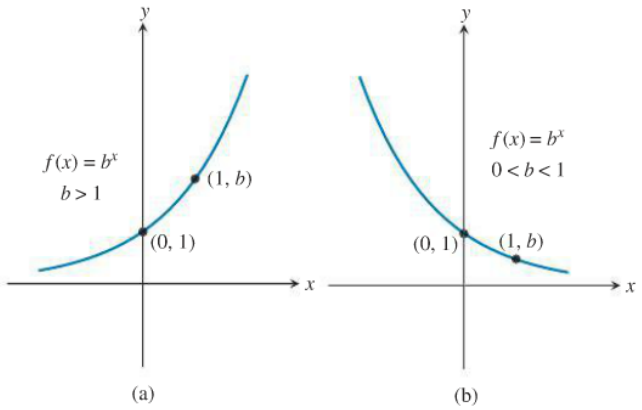

FIGURE 3.3 Graphs of $f(x) = b^x$ for (a) $b > 1$ and (b) $0 < b < 1$.

Domain: $(-\infty, \infty)$
Range: $(0, \infty)$
Continuous
No symmetry: neither even nor odd
Bounded below, but not above
No local extrema
Horizontal asymptote: $y = 0$
No vertical asymptotes
If $b > 1$ (see Figure 3.3a), then

- f is an increasing function,
- $\lim_{x \to -\infty} f(x) = 0$ and $\lim_{x \to \infty} f(x) = \infty$.

If $0 < b < 1$ (see Figure 3.3b), then

- f is a decreasing function,
- $\lim_{x \to -\infty} f(x) = \infty$ and $\lim_{x \to \infty} f(x) = 0$.

In Exercises 1–6, which of the following are exponential functions? For those that are exponential functions, state the initial value and the base. For those that are not, explain why not.

1. $y = x^8$
2. $y = 3^x$
3. $y = 5^x$
4. $y = 4^2$
5. $y = x^{\sqrt{x}}$
6. $y = x^{1.3}$

In Exercises 7–10, compute the exact value of the function for the given x-value without using a calculator.

7. $f(x) = 3 \cdot 5^x$ for $x = 0$
8. $f(x) = 6 \cdot 3^x$ for $x = -2$
9. $f(x) = -2 \cdot 3^x$ for $x = 1/3$
10. $f(x) = 8 \cdot 4^x$ for $x = -3/2$

In Exercises 11 and 12, determine a formula for the exponential function whose values are given in Table 3.6.

11. $f(x)$
12. $g(x)$

Table 3.6 Values for Two Exponential Functions

x	$f(x)$	$g(x)$
-2	6	108
-1	3	36
0	3/2	12
1	3/4	4
2	3/8	4/3

Graph the function and analyze it for domain, range, increasing or decreasing behavior, boundedness, extrema, asymptotes and end behavior.

$$f(x) = 3\left(\frac{1}{4}\right)^x$$

DO NOT USE A CALCULATOR FOR THIS PROBLEM!

x	-3	-2	-1	0	1	2
f(x)						

EXPLORATION 1 Graphs of Exponential Functions

1. Graph each function in the viewing window $[-2, 2]$ by $[-1, 6]$.

(a) $y_1 = 2^x$ **(b)** $y_2 = 3^x$ **(c)** $y_3 = 4^x$ **(d)** $y_4 = 5^x$

- Which point is common to all four graphs?
- Analyze the functions for domain, range, continuity, increasing or decreasing behavior, symmetry, boundedness, extrema, asymptotes, and end behavior.

2. Graph each function in the viewing window $[-2, 2]$ by $[-1, 6]$.

(a) $y_1 = \left(\frac{1}{2}\right)^x$ **(b)** $y_2 = \left(\frac{1}{3}\right)^x$

(c) $y_3 = \left(\frac{1}{4}\right)^x$ **(d)** $y_4 = \left(\frac{1}{5}\right)^x$

- Which point is common to all four graphs?
- Analyze the functions for domain, range, continuity, increasing or decreasing behavior, symmetry, boundedness, extrema, asymptotes, and end behavior.